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Engineering Notes

ENGINEERING NOTES are short manuscripts describing new developments or important results of a preliminary nature. These Notes cannot exceed six manuscript pages and three figures; a page of text may be substituted for a figure and vice versa. After informal review by the editors, they may be published within a few months of the date of receipt. Style requirements are the same as for regular contributions (see inside back cover).

Operating Mode of Small-Scale Arc Heaters

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Nomenclature

A = arc-heater channel area, m^2
A_r = restricted area, m^2
E = electric field, $V\ m^{-1}$
h = gas specific enthalpy, $J\ kg^{-1}$
I = arc current intensity, A
j = current density, $A\ m^{-2}$
k_{SB} = Stefan–Boltzmann constant, $W\ m^{-2}\ K^{-4}$
$\dot{m}$ = gas mass flow rate, $kg\ s^{-1}$
p = gas pressure, Pa
R = arc-heater channel radius, m
r = radial coordinate, m
T = gas temperature, K
V = electric potential, V
v = gas velocity, $m\ s^{-1}$
w_{rad} = radiated heat, W
x = axial coordinate, m
α_G = gas absorbance
γ = gas specific heat ratio
ε = arc-heater efficiency
ε_G = gas emittance
κ = gas thermal conductivity, $W\ m^{-1}\ K^{-1}$
π_1, π_2 = arc-heater nondimensional parameters
ρ = gas density, $kg\ m^{-3}$
σ = gas electric conductivity, $\Omega^{-1}\ m^{-1}$

Subscripts

in = inflow conditions
out = outflow conditions
wall = wall conditions
$(\bar{\ })$ = nondimensional variable

Introduction

ARC heaters have been used since the late 1950s as a convenient method for generating high temperature gas streams for hypersonic simulation purposes.[1,2] In this particular type of application, for given geometric, electrical, and flow characteristics, the relevant operating parameter is the enthalpy increase of the gas flowing across the heater, $\Delta h = (h_{out} - h_{in})$, which provides us with a simple way to define the gas heater efficiency:

$$\varepsilon = \frac{\dot{m}\Delta h}{I \Delta V} \tag{1}$$

that is, the ratio between the power transferred to the fluid flow and the power dissipated at the heater electrodes. The efficiency is usually quite low (on the order of 0.3–0.7) because of two main causes: direct thermal losses (by conduction or radiation to the heater walls[3]) and losses caused by electrode processes (voltage drop across the sheaths[4]).

Because of the technological interest of such devices, much effort has been spent in developing engineering methods for sizing them; nevertheless, since the early pioneering works,[3,5] the attention has focused only on the constricted-type arc heater (Huels or segmented). In such a heater, a relatively long arc column is established between cathode and anode separated by a long constricted channel[6]: for these configurations, the classical arc theory approach, i.e., neglecting axial gradients with respect to radial ones, has been quite successful in predicting operating mode and capability.

Since 1996, a small-scale arc heater (HEAT) has been operated at Centrospazio to run a pulsed, quasi-steady Mach 6 wind tunnel.[7–9] The heater has been demonstrated at different power levels (between 30 and 150 kW at the electrodes, with efficiency ranging between 0.4 and 0.5) and proved to be a reliable and flexible tool for performing aerothermodynamic tests. The heater design, however, is considerably different from those usually considered in the previous references. Indeed, because of the small arc column diameter-to-length ratio, classical arc theory solutions are not directly applicable to this type of device. For this reason, a different approach was investigated for the purpose of providing scaling information useful for sizing and preliminary analysis.

Arc-Heater Model Equation

A simplified model for the behavior of an arc column can be derived from the energy balance equation, which for a conductive inviscid fluid, flowing steadily along a constant section channel, can be written as

$$\nabla \cdot (\rho \boldsymbol{v} h) - \nabla \cdot (\kappa \nabla T) + w_{rad} = j^2/\sigma \tag{2}$$

where the current density is expressed by Ohm's law:

$$\boldsymbol{j} = \sigma \boldsymbol{E} \tag{3}$$

and the electric field is related to the voltage drop as follows:

$$\Delta V = \int_l \boldsymbol{E} \cdot \mathrm{d}\boldsymbol{l} \tag{4}$$

In cylindrical coordinates and neglecting axial conduction and radial convection, this is simplified as

$$\frac{\partial}{\partial x}(\rho v h) - \frac{1}{r}\frac{\partial}{\partial r}\left(r\kappa\frac{\partial T}{\partial r}\right) + w_{rad} = \frac{j^2}{\sigma} \tag{5}$$

which becomes the well-known Elenbaas–Heller (E–H) equation if we further neglect axial gradients (which is equivalent to assuming an infinitely long arc column) and radiative heat transfer[10]:

$$-\frac{1}{r}\cdot\frac{\mathrm{d}}{\mathrm{d}r}\left(r\cdot\kappa\frac{\mathrm{d}T}{\mathrm{d}r}\right) = \frac{j^2}{\sigma} \tag{6}$$

It is interesting to note that the E–H model equation can be solved in closed form if we assume that the thermal and electric conductivity

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are the function of neither the temperature nor the position, yielding

$$T = T_{\text{wall}} + j^2 R^2 / 4\sigma\kappa \tag{7}$$

In the case of an electric arc in air, causing a moderate bulk enthalpy increase in a steady wall-bounded flow, a plausible functional dependence can be assumed for the axial velocity component and the radiated heat. For the former, we have from one-dimensional gasdynamics $v = \dot{m}/\rho A$, where the mass flow is controlled by the upstream restricted section[9]:

$$\dot{m} = \left(pA_r \big/ \sqrt{T}\right) f(\gamma, p) \tag{8}$$

The radiated heat transfer from a hot gas to the enclosing vessel can be approximated as follows[11]:

$$w_{\text{rad}} \propto k_{\text{SB}}\left(\varepsilon_G T^4 - \alpha_G T_{\text{wall}}^4\right) \tag{9}$$

where both ε_G and α_G are functions of the gas pressure and temperature. The functional dependence can, therefore, be summarized as $v = v(p, T)$ and $w_{\text{rad}} = w_{\text{rad}}(p, T)$. In addition, when the electric field is parallel to both the fluid and current flow, the vector equation for the electric potential can be simplified as $E = \mathrm{d}V/\mathrm{d}x$.

The energy balance equation for a steady, wall-bounded, arc-heated flow can, therefore, be written as follows:

$$\frac{\partial}{\partial x}[\rho h v(p, T)] - \frac{1}{r}\frac{\partial}{\partial r}\left(r\kappa\frac{\partial T}{\partial r}\right) + w_{\text{rad}}(p, T) = j\,\frac{\mathrm{d}V}{\mathrm{d}x} \tag{10}$$

or, by integrating over the arc column cross section:

$$\frac{\partial}{\partial x}[h\dot{m}(p, T)] - \int_S \left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\kappa\frac{\partial T}{\partial r}\right) + w_{\text{rad}}(p, T)\right)\mathrm{d}S = I\,\frac{\mathrm{d}V}{\mathrm{d}x} \tag{11}$$

Dimensional Analysis

Although extremely simple, the E–H model [Eq. (6)] proved to be very useful to provide qualitative, and sometimes also quantitative, information about the complex thermal phenomena occurring within an arc heater. Unfortunately, as previously mentioned, the model cannot be directly applied to the short-arc configuration characterizing HEAT. The generalized E–H model [Eq. (11)] does, however, identify a few relevant variables that are the driving parameters of the phenomenon. This information is sufficient to apply some of the tools provided by dimensional analysis, and in particular Buckingham's π theorem.

In the present case, we have five fundamental dimensions (length, mass, time, temperature, and electric charge) and seven relevant physical parameters (pressure, density, temperature, thermal conductivity, current intensity, voltage drop, and enthalpy variation). From the π theorem, we can determine two nondimensional relations linking these variables. It is easily verified that these new parameters can be written in the form

$$\pi_1 = \frac{(\Delta h)^{0.75}\rho\kappa T}{p^{1.5}(VI)^{0.5}}, \qquad \pi_2 = \frac{\kappa T}{(pVI)^{0.5}(\Delta h)^{0.25}} \tag{12}$$

To highlight the significance of such newly defined nondimensional parameters, it should be noted that they can be used to make the E–H model equation nondimensional by applying suitable reference values. It can be verified by substitution that the nondimensional equation is as follows:

$$-\frac{1}{\bar{r}}\cdot\pi_1\pi_2\cdot\frac{\mathrm{d}}{\mathrm{d}\bar{r}}\left(\bar{r}\cdot\bar{\kappa}\frac{\mathrm{d}\bar{T}}{\mathrm{d}\bar{r}}\right) = \bar{j}\bar{E} \tag{13}$$

Experimental Data Reduction

A large set of measured arc-heater operating parameters, recorded during the years 1996–1998 for more than 150 tunnel runs at different power levels, was used to build a reference database in terms of the nondimensional numbers π_1 and π_2, computed using the pressure and density values just downstream of the arc region, the wall temperature, the thermal conductivity corresponding to the wall temperature, the arc current and voltage drop, and the effective flowing gas enthalpy jump across the arc region.

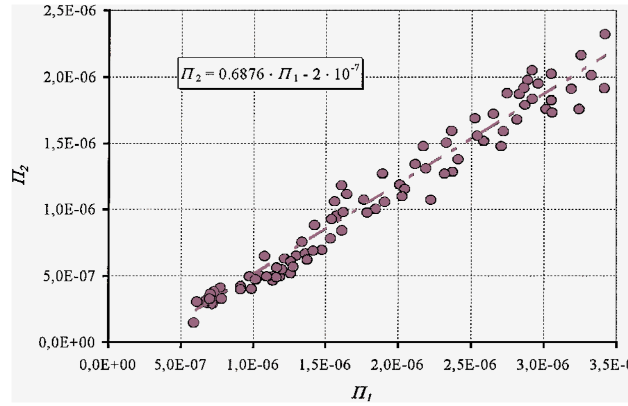

Fig. 1 Experimental data plotted on the π_1–π_2 plane.

As shown in Fig. 1, when plotted on the π_1–π_2 plane the operating points tend to collapse on a line characterized by the following equation:

$$\pi_2 = 0.6876\cdot\pi_1 - 2.0\times10^{-7} \tag{14}$$

It should be noted that, although the correlation is fair over the whole range of the tested power levels, data points corresponding to highly energetic operation are rather dispersed, indicating that other parameters may play an important role at the more extreme conditions. However, the conclusion that can be drawn from the collected data is that Eq. (11) is capturing most of the relevant physics in the arc region, at least for the type of electric discharge which takes place in the HEAT arc-heater. The model equation [Eq. (11)], together with Eq. (12) and Eq. (14), is providing a useful tool for the preliminary analysis and sizing of small, pulsed arc heaters.

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